Electromagnetic instabilities in unmagnetized plasmas

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It is shown that local perturbations in nonuniform unmagnetized plasmas can give rise to linearly growing electromagnetic fields on both electron and ion time scales. The plasma vorticity and compressibility couple due to density inhomogeneity, giving rise to instabilities with partially transverse and partially longitudinal characteristics in electron plasmas for $\omega_{pe} \ll \omega$ and in electron-ion plasmas for $\omega_{pe} \ll \omega$ as well as $\omega \ll \omega_{pe}$ (where ω_{pe} is the electron plasma oscillation frequency). It is reconfirmed that the pure transverse modes due to the thermoelectric term do not appear in nonuniform unmagnetized electron plasmas. Furthermore, it has been found that the thermal fluctuations in a collisionless inhomogeneous electron plasma happen on a slower time scale of the order of $1/c_s k$ (where c_s is the ion sound speed). It seems that in the presence of a steep density gradient the ion acoustic wave becomes electromagnetic. Since the curl of electric field becomes nonvanishing in the presence of a density gradient, any nonuniform plasma can have magnetic field fluctuations in the limit $\omega \ll \omega_{pe}$ as well. It is suggested that in the limit $\omega \ll \omega_{pe}$ the ion dynamics becomes important and a pure electron plasma model to study magnetic field instability is not useful. The estimate for the magnitude of slowly and rapidly growing magnetic fields using the electron-ion plasma model in a special range of parameters turns out to be of the order of megagauss, in good agreement with the experimental observations.

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I. INTRODUCTION

Large magnetic fields (of the order of megagauss) have been observed in laser produced plasmas for some time [1–5]. Several mechanisms have been proposed to understand the cause of generation of these magnetic fields [6–20]. Most of the theoretical investigations in this direction consider electron temperature perturbations to be important for the unstable pure transverse waves in an inhomogeneous plasma, assuming the ions to be stationary. It is generally believed that this mechanism of magnetic field generation is fast enough and even in the low-frequency limit ion dynamics is not important. At the same time in the low-frequency limit the electron displacement current is ignored, which gives zero electron density perturbation. Much research on magnetic field generation is based on the thermoelectric term $\nabla n_0 \times \nabla T_1$ (where ∇n_0 is the equilibrium density gradient and T_1 is the linear electron temperature perturbation). In a collisionless electron plasma the low-frequency purely transverse perturbations are generally studied in the range $\omega_{pi} \ll \omega \ll \omega_{pe}$ [where ω_{pe} (ω_{pi}) is the electron (ion) plasma oscillation frequency]. The oscillatory magnetic instability in the high-frequency limit $\omega_{pe} \ll \omega$ due to electron temperature perturbations has also been considered [15]. These low- and high-frequency perturbation models have been applied to the plasmas of low-Z materials like hydrogen and its isotopes, assuming the ions to be stationary.

Recently it has been shown that in a collisionless plasma the thermoelectric term does not become a source for magnetic fluctuations in the linear limit [19]. Furthermore, the assumptions of divergence-free electron current and stationary ions do not present a realistic situation at least in a low-Z material plasma within the framework of local theory, which requires $\kappa_n \ll k$ (where κ_n is the inverse of the density gradient scale length L_n). Similar assumptions are used in electron magnetohydrodynamics which also presents contradictions [21]. It has also been pointed out that temperature perturbations cannot give rise to pure transverse instabilities [19]. The assumptions used to describe an incompressible thermal wave with $\omega_{pi} \ll \omega \ll \omega_{pe}$, $\nabla \cdot \mathbf{v}_1 \neq 0$ and $\nabla \cdot \mathbf{j}_1 = 0$ (where $\mathbf{v}_1$ and $\mathbf{j}_1$ are the linear velocity and current perturbations, respectively) do not seem to be self-consistent.

In the present paper we discuss two plasma slab models. We show that a linear perturbation can give rise to electromagnetic instabilities having partially longitudinal and partially transverse characteristics in inhomogeneous unmagnetized pure electron plasmas as well as electron-ion plasmas. Numerical values of ω in the coupled dispersion relation in a pure electron plasma show that the electron temperature perturbation (with $\omega < \omega_{pe}$) corresponds to an instability on the ion time scale with $\omega \sim c_s k$ (where c_s is the ion sound speed). Therefore, we must consider ion dynamics as well in this limit.

On the other hand, in inhomogeneous electron-ion plasmas a linear perturbation presents a coupling of high-frequency plasma waves, ordinary transverse waves, and low-frequency electromagnetic waves near the ion acoustic frequency [19]. Numerically calculated ω values of the coupled dispersion relation show that a rapidly fluctuating magnetic field (with $\omega_{pe} \ll \omega$) and a slowly oscillating field (with $\omega \sim c_s k$) can grow simultaneously in an electron-ion inhomogeneous plasma.

In both the above inhomogeneous unmagnetized plasma models the steady state is assumed to be maintained by external forces like laser beams. Apart from this assumption these models are general and just show that a linear perturbation in a nonuniform plasma slab can give rise to growing magnetic fluctuations on both ion and electron time scales with real frequencies mainly near $c_s k$ and ω_{pe}.

In the next section we study the linear perturbations in an electron plasma including temperature fluctuations. In Sec. III the coupling of transverse and longitudinal modes is investigated in an electron-ion plasma. Section IV gives some rough estimates of the order of magnitude of the generated magnetic fields. Finally, in Sec. V we discuss these plasma models and the numerical results in some detail.

II. PERTURBED ELECTRON PLASMA

We consider a pure electron plasma with stationary ions having density and temperature gradients along the x axis. There are no external electric or magnetic fields and the unperturbed current is zero in the static plasma. The set of basic equations used is as follows: the equation of motion

$$mn(\partial_t+\mathbf{v}\cdot\nabla)\mathbf{v}=-en\mathbf{E}-\nabla p, \tag{1}$$

the continuity equation

$$\partial_t n+\nabla\cdot(n\mathbf{v})=0, \tag{2}$$

the Poisson equation

$$\nabla\cdot\mathbf{E}=-4\pi en, \tag{3}$$

the adiabatic temperature equation

$$\tfrac{3}{2}n(\partial_t+\mathbf{v}\cdot\nabla)T+p\nabla\cdot\mathbf{v}=0. \tag{4}$$

The equation of state is assumed to be the ideal gas law $p=nT$. In addition to these we need electromagnetic equations, Faraday's law

$$\nabla\times\mathbf{E}=-\frac{1}{c}\partial_t\mathbf{B}, \tag{5}$$

and Ampère's law

$$\nabla\times\mathbf{B}=\frac{4\pi}{c}\mathbf{j}+\frac{1}{c}\partial_t\mathbf{E}. \tag{6}$$

All of the variables used here have their standard meaning. The curls of the equation of motion and Ampère's law give, respectively,

$$\partial_t\nabla\times(n_0\mathbf{v}_1)=\frac{-e}{m}\nabla\times(n_0\mathbf{E}_1) \tag{7}$$

and

$$(\partial_t^2-c^2\nabla^2)\mathbf{B}_1=-4\pi ec\nabla\times(n_0\mathbf{v}_1), \tag{8}$$

where the subscripts 0 and 1 denote the background and perturbed quantities, respectively. It is important to note that the pressure term disappears and does not contribute to vorticity directly. Assuming the perturbations to be proportional to $\exp i(\mathbf{k}\cdot\mathbf{r}-\omega t)$, Eqs. (7) and (8) give

$$(\partial_t^2-c^2\nabla^2)\mathbf{B}_1=\frac{i\omega_{pe}^2}{\omega}c(i\mathbf{k}+\boldsymbol{\kappa}_n)\times\mathbf{E}_1, \tag{9}$$

where $\boldsymbol{\kappa}_n=\kappa_n\mathbf{x}$ and $\kappa_n=(1/n_0)(dn_0/dx)$.

If a pure transverse perturbation is considered with $\mathbf{k}=k\mathbf{x}$, $\mathbf{E}_1=-[(1/c)\partial_t A_{1y}]\mathbf{y}$ satisfying $\nabla\cdot\mathbf{E}_1=0$, then Eq. (9) becomes

$$[\omega^2-c^2k^2-\omega_{pe}^2(1-iq_n)]A_{1y}=0, \tag{10}$$

where $q_n=k_x\kappa_n/k^2$. However, we expect that the transverse and longitudinal modes can couple due to inhomogeneity as Eq. (9) suggests. Therefore, we take into account the electrostatic potential perturbation ϕ_1 as well by defining

$$\mathbf{E}_1=-\nabla\phi_1-\frac{1}{c}\partial_t\mathbf{A}_1, \tag{11}$$

and for the sake of generality we consider $\mathbf{k}=(k_x,k_y,0)$ and $\mathbf{A}_1=(A_{1x},A_{1y},0)$ with Coulomb gauge $\nabla\cdot\mathbf{A}_1=0$. In this case Eq. (9) becomes

$$[\omega^2-c^2k^2-\omega_{pe}^2(1-iq_n)]A_{1y}=i\frac{\omega_{pe}^2}{\omega}ck_yq_n\phi_1, \tag{12}$$

where $\mathbf{B}_1=\nabla\times\mathbf{A}_1$ has been used. Equation (12) suggests that the vorticity and compressibility can couple in nonuniform plasmas. Actually the longitudinal plasma waves and transverse ordinary waves can couple because $\nabla n_0\neq\mathbf{0}$ in Eq. (12). Both are high-frequency modes with $\omega_{pe}\leqslant\omega$. Electron temperature perturbations can also take place because the compressibility $\nabla\cdot\mathbf{v}_1\neq0$. Later we shall notice that electron thermal waves have a frequency near ion acoustic modes $\omega\sim c_sk$ and do not contribute to magnetic fluctuations on the electron time scale. Therefore, in electron plasmas a transverse mode with $\omega_{pi}\ll\omega\ll\omega_{pe}$ cannot exist because $T_1\neq0$.

To further eleborate this point we retain T_1 and solve the coupled dispersion relation analytically. Equation (1) gives

$$\mathbf{v}_1=-\frac{i}{\omega}\left(\frac{e}{m}\mathbf{E}_1+v_{te}^2\mathbf{g}_0\frac{n_1}{n_0}+\frac{1}{m}\mathbf{g}_2T_1\right) \tag{13}$$

and

$$\nabla\cdot\mathbf{v}_1=-\frac{i}{\omega}\left(\frac{e}{m}\nabla\cdot\mathbf{E}_1+v_{te}^2(\boldsymbol{\kappa}_T+\mathbf{g}_1)\cdot\mathbf{g}_0\frac{n_1}{n_0}+v_{te}^2(i\mathbf{k}\cdot\mathbf{g}_2)\frac{T_1}{T_0}\right), \tag{14}$$

where $\mathbf{g}_0=i\mathbf{k}+\boldsymbol{\kappa}_T$, $\boldsymbol{\kappa}_T=\kappa_T\hat{x}$, $\kappa_T=(1/dT_0)(dT_0/dx)$, $\mathbf{g}_1=i\mathbf{k}-\boldsymbol{\kappa}_n$, and $\mathbf{g}_2=i\mathbf{k}+\boldsymbol{\kappa}_n$. The continuity equation gives

$$\frac{n_1}{n_0}=-\frac{1}{\alpha}\left(\frac{e}{m}\mathbf{g}_2\cdot\mathbf{E}_1+v_{te}^2\mathbf{g}_2^2\frac{T_1}{T_0}\right), \tag{15}$$

where $\alpha=\omega^2+v_{te}^2g_0^2$. Using Eq. (15) in the Poisson equation, we can obtain the relation between temperature and electric field as

$$\frac{T_1}{T_0}=\frac{1}{4\pi n_0 e v_{te}^2 g_2^2}[\alpha\nabla\cdot\mathbf{E}_1-\omega_{pe}^2\mathbf{g}_2\cdot\mathbf{E}_1]. \tag{16}$$

Equation (4) can be simplified as

$$(\tfrac{3}{2}\omega^2+v_{te}^2\mathbf{g}_2\cdot\mathbf{g}_3)\frac{T_1}{T_0}=-\left(\frac{e}{m}\mathbf{g}_3\cdot\mathbf{E}_1+v_{te}^2\mathbf{g}_0\cdot\mathbf{g}_4\frac{n_1}{n_0}\right), \tag{17}$$

where $\mathbf{g}_3=i\mathbf{k}+\frac{3}{2}\boldsymbol{\kappa}_T$. Substituting n_1 from Eq. (15) into Eq. (17), we obtain another relation between T_1 and $\mathbf{E}_1$, viz.,

$$\left(\frac{3}{2}\omega^2+v_{te}^2\mathbf{g}_2\cdot\mathbf{g}_3-\frac{v_{te}^2\mathbf{g}_0\cdot\mathbf{g}_4}{\alpha}v_{te}^2g_2^2\right)\frac{T_1}{T_0}=-\frac{e}{m}\left[\mathbf{g}_3\cdot\mathbf{E}_1-\frac{v_{te}^2\mathbf{g}_0\cdot\mathbf{g}_4}{\alpha}\mathbf{g}_2\cdot\mathbf{E}_1\right], \tag{18}$$

where $\mathbf{g}_4=\mathbf{g}_1+\frac{5}{2}\kappa_T$. Then Eqs. (16) and (18) give an equation in $\mathbf{E}_1$ as follows:

$$\tfrac{3}{2}(\alpha\nabla\cdot\mathbf{E}_1-\omega_{pe}^2\mathbf{g}_2\cdot\mathbf{E}_1)\omega^2-\omega_{pe}^2[v_{te}^2(\mathbf{g}_2\cdot\mathbf{g}_3)\mathbf{g}_2\cdot\mathbf{E}_1-v_{te}^2g_2^2\mathbf{g}_3\cdot\mathbf{E}_1]-[v_{te}^4g_2^2(\mathbf{g}_0\cdot\mathbf{g}_4)-\alpha v_{te}^2(\mathbf{g}_2\cdot\mathbf{g}_3)](\nabla\cdot\mathbf{E}_1)=0. \tag{19}$$

Inserting E_1 from Eq. (11) into Eq. (19) we obtain a relation between electrostatic and magnetic vector potentials, viz.,

$$\{\tfrac{3}{2}\omega^4-[\tfrac{3}{2}\omega_{pe}^2(1-iq_n)+\tfrac{5}{2}v_{te}^2k^2-(T_{2r}+iT_{2i})]\omega^2-(T_{0r}+iT_{0i})\}\phi_1=-\frac{k_y}{k_x}[\tfrac{3}{2}\omega_{pe}^2\mu_n\omega^2+(S_r+iS_i)]\frac{i\omega}{ck}A_{1y}, \tag{20}$$

where

$$T_{2r}=\frac{3}{2}v_{te}^2k^2\mu_T(\mu_T+\mu_n),$$

$$T_{2i}=v_{te}^2k^2\left(4q_n+\frac{3}{2}q_T\right),$$

$$T_{0r}=\omega_{pe}^2v_{te}^2k_y^2\mu_n\left(\frac{3}{2}\mu_T-\mu_n\right)+v_{te}^4k^4G_r,$$

$$T_{0i}=v_{te}^4k^4G_i,$$

$$S_r=\omega_{pe}^2v_{te}^2k^2\left(\frac{3}{2}\mu_T-\mu_n\right),$$

$$S_i=-\omega_{pe}^2v_{te}^2k^2\left(\frac{3}{2}\mu_T-\mu_n\right)q_n,$$

$$G_r=-\frac{3}{2}\mu_T^2+\mu_T\mu_n\left(\frac{5}{2}+\frac{5}{2}\mu_T\mu_n-\mu_n^2-\frac{3}{2}\mu_T^2\right)+4q_n(q_n-q_T)-\mu_n^2,$$

$$G_i=2(q_n-q_T)-5\mu_T\mu_nq_n+\mu_T^2\left(4q_n-\frac{3}{2}q_T\right)-\mu_n^2\left(q_n-\frac{7}{2}q_T\right),$$

$$\mu_n=\frac{\kappa_n}{k},\quad \mu_T=\frac{\kappa_T}{k},\quad \text{and}\quad q_T=\frac{k_x}{k}\mu_T.$$

The terms with coefficients T and S in Eq. (20), having corresponding subscripts, represent the contributions of thermal fluctuations. If these terms are dropped then Eq. (20) represents linear coupling of electrostatic plasma waves with electromagnetic ordinary waves through $A_{1y}\neq 0$ and $\mu_n\neq 0$. In this case Eqs. (12) and (20) give

$$[\omega^2-\omega_{pe}^2(1-iq_n)-\tfrac{5}{3}v_{te}^2k^2][\omega^2-c^2k^2-\omega_{pe}^2(1-iq_n)]=\left(\frac{k_y\kappa_n}{k^2}\right)^2\omega_{pe}^4. \tag{21}$$

Let $L_{es}^2=\omega_{pe}^2(1-iq_n)+\gamma_e v_{te}^2k^2$, $L_{em}^2=\omega_{pe}^2(1-iq_n)+c^2k^2$, and $q_y=k_y\kappa_n/k^2=(k_y/k_x)q_n$ so that Eq. (21) can be written as

$$\left(\frac{\omega^2-L_{es}^2}{\omega_{pe}^2}\right)\left(\frac{\omega^2-L_{em}^2}{\omega_{pe}^2}\right)=q_y^2. \tag{22}$$

The coupling presented in Eq. (22) is possible if $m/M\ll q_y^2$ so that the neglect of ion dynamics is justified. Combined with the local approximation this becomes

$$\left(\frac{m}{M}\right)^{1/2}\ll q_y\ll 1. \tag{23}$$

Such a coupling of high-frequency transverse and longitudinal waves seems to take place in a steep density gradient electron plasma within a very narrow range of wavelengths. When ion dynamics is also considered then this restriction is relaxed. In that case the coupling predicted by Eq. (21) can take place in a larger range of wavelengths in principle. If the thermal fluctuations are also taken into account, then Eqs. (12) and (20) yield a sixth order polynomial in ω, viz.,

$$\tfrac{3}{2}\omega^6+[(b-\tfrac{3}{2}a)+i(d+\tfrac{3}{2}q_n)]\omega_{pe}^2\omega^4-\left[\left(ab+q_nd+Y_r+\frac{3}{2}\frac{k_y^2}{k_xk}q_n\mu_n\right)+i(ad-q_nb+Y_i)\right]\omega_{pe}^4\omega^2+[(aY_r+q_nY_i-h)$$
$$+i(aY_i-q_nY_r+q_nh)]\omega_{pe}^6=0, \quad (24)$$

where

$$a=1+\lambda^2k^2,$$

$$b=\left\{-\frac{3}{2}+\lambda_{De}^2k^2\left[-\frac{5}{2}+\frac{3}{2}\mu_T(\mu_T+\mu_n)\right]\right\},$$

$$d=\frac{3}{2}q_n+\lambda_{De}^2k^2\left(4q_n+\frac{3}{2}q_T\right),$$

$$h=\frac{k_y^2}{k_xk}\lambda_{De}^2k^2q_n\left(\frac{3}{2}\mu_T-\mu_n\right),$$

$$Y_r=\lambda_{De}^2k^2G_r+\lambda_{De}^2k_y^2\mu_n\left(\frac{3}{2}\mu_T-\mu_n\right),$$

$$Y_i=\lambda_{De}^4k^4G_i.$$

As an illustration we solve Eq. (24) numerically for two examples of laser-produced D-D plasmas. Let example 1 be $n_0\sim10^{20}$ cm^{-3}, $T_0\sim100$ eV [17], and example 2 be $n_0\sim10^{22}$ cm^{-3}, $T_0\sim$ 1 keV [16]. There are six complex conjugate roots of this dispersion relation. Three of them turn out to be growing in numerical solutions. This necessitates a few comments.

First, one of the complex roots corresponds to the longitudinal plasma wave that looks artificially unstable in this hydrodynamic model. It is well known that this wave suffers from Landau damping due to wave-particle interaction in both the long and short wavelength limts, $\lambda_{De}k\ll1$ or $1\ll\lambda_{De}k$ [22]. We are using fluid theory and hence work within the limit $\lambda_{De}k\ll1$. If the kinetic model is used the plasma wave will be damped even in an inhomogeneous plasma due to wave-particle interaction. Some interesting limits on the plasma and perturbation parameters may also appear for the instability of high-frequency transverse modes. However, this aspect is beyond the scope of the present investigation.

Second, the real frequency of the electron thermal wave in nonuniform plasmas turns out to be near the ion acoustic frequency. Therefore we discard these solutions in the context of the electron plasma.

Third, in the stationary ion case only the high-frequency transverse wave becomes unstable due to density inhomogeneity. The linear growth rate is very large and therefore the nonlinearities can come into play very quickly. Nonlinear study of such a perturbation may give an increasing magnetic field on a slow time scale as well. However, within the linear limits we do not find a growing magnetic perturbation with $\omega\ll\omega_{pe}$ in a stationary ion plasma. The numerical solutions of the coupled dispersion relation Eq. (24) show that the thermal wave has a real frequency of the order of $\omega_r\sim c_sk$ and therefore ion dynamics cannot be ignored if we take into account electron thermal fluctuations. Furthermore, the growth rates do not depend much on the temperature gradient. We notice that such a magnetic instability cannot be suppressed even with a uniform compression ($\nabla p_0=0$) of the plasma.

Figures 1(a) and 1(b) show the dependence of real frequency ω_r and imaginary frequency ω_i upon the wave number k for both the examples 1 and 2. The growth rates of the high-frequency electromagnetic waves are very large and hence nonlinearities appear to be very important.

III. PERTURBED ELECTRON-ION PLASMA

Here we study linear perturbations in an unmagnetized nonuniform electron-ion plasma without ignoring the electron inertia. For simplicity we assume electrons to be adiabatic ($\nabla p_{e1}\sim\gamma T_{e0}\nabla n_{e1}$) with the local approximation, and ions to be cold. In Ref. [19] only the low-frequency electromagnetic instability was investigated. In the limit $\omega\ll\omega_{pe}$ the electrons were assumed to be isothermal. In the present study we solve the coupled sixth order dispersion relation

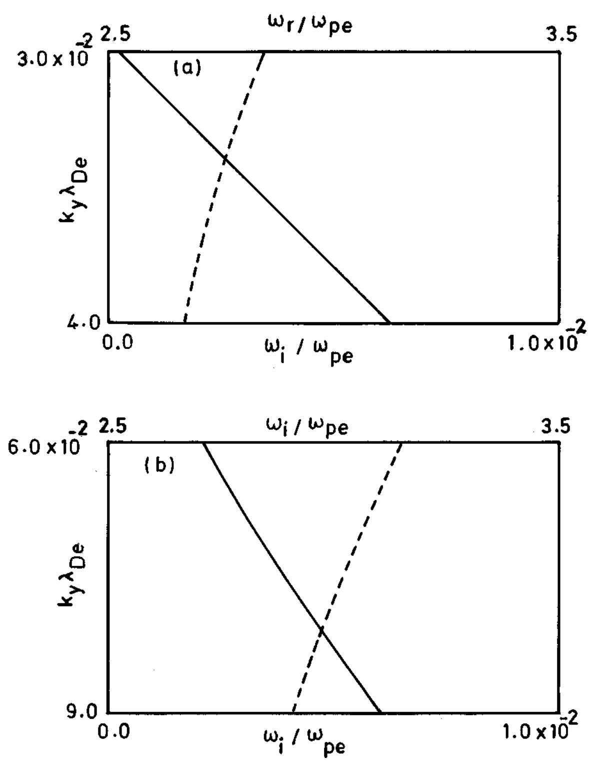


FIG. 1. The real ω_r (solid curve) and imaginary ω_i (dashed curve) frequencies of fast electromagnetic waves in pure electron plasmas are plotted (a) for example 1 with $\kappa_n\sim5\times10^3$ cm^{-1} and $k_x\sim4\times10^4$ cm^{-1}, (b) for example 2 with $\kappa_n\sim2\times10^4$ cm^{-1} and $k_x\sim4\times10^5$ cm^{-1}.

and find that electromagnetic instabilities can occur on both electron and ion time scales. Since the electrons are considered to be adiabatic, $v_{te}^2 \approx \gamma_e T_0/m_e$ and $\gamma = \frac{5}{3}$ for the ideal gas.

Assuming the same perturbation geometry as that used in the case of an electron plasma with the electric field of Eq. (11), the coupling of compressibilities and vorticities of electron and ion fluids yields

$$\{[\omega^2-(1-iq_n)\omega_{pi}^2](\omega^2-v_{te}^2k^2)-(1-iq_n)\omega_{pe}^2\omega^2\}[\omega^2-c^2k^2-(1-iq_n)\omega_{ei}^2]-\left(\frac{\kappa_n k_y}{k^2}\right)^2\omega_{ei}^2(\omega^2\omega_{ei}^2-\omega_{pi}^2v_{te}^2k^2)=0, \quad (25)$$

where $\omega_{ei}^2=\omega_{pe}^2+\omega_{pi}^2$. Equation (25) is the same as Eq. (19) of Ref. [19] which gives a stable low-frequency electromagnetic mode in the limit $\omega \ll \omega_{pe}$ with $\omega_r \sim c_s k$ and $k_x=0$. This mode can become unstable if $k_x \neq 0$. Since it is not a purely electrostatic mode we avoid calling it a modified ion acoustic mode. If the ion contribution is dropped using $\omega_{pi} \ll \omega_{pe}$, then Eq. (25) reduces to Eq. (21).

We note that if we study the linear perturbations in an inhomogeneous electron-ion plasma in a more natural way with $m/M \neq 0$, there appears an electromagnetic wave near $\omega \sim c_s k$. Furthermore, in the presence of a density gradient the high-frequency ordinary waves can couple with these low-frequency extraordinary modes. The free energy stored in the form of the density gradient can enhance the electromagnetic fields due to plasma convection. Both the high- and low-frequency electromagnetic waves can become unstable. We note also that for the same plasma parameters as have been used in the two examples of electron plasmas and for similar perturbation wavelengths the high-frequency and low-frequency electromagnetic waves can couple if $m/M \neq 0$ is used in electron-ion plasmas.

Figures 2(a) and 2(b) show the instabilities of linear electromagnetic perturbations on ion time scales in examples 1 and 2 of laser-produced plasmas with $\omega_r \sim c_s k$ and $\omega_i \ll c_s k$. The high-frequency wave in electron-ion plasmas has a similar behavior to that found for the case of pure electron plasmas in Figs. 1(a) and 1(b), but in the electron-ion plasma both the high-frequency and low-frequency electromagnetic modes are coupled and grow on electron and ion time scales.

IV. ESTIMATE OF B FIELD MAGNITUDES

Direct estimation of the magnitudes of the magnetic fields produced as a result of linear perturbations is difficult. However, if we express the magnetic field in terms of the electrostatic potential then we can make some estimate of the magnitudes by assuming some initial electrostatic energy fluctuation relative to the plasma thermal energy, which is given by T_0. Let $\omega=\omega_r+i\omega_i$ (where ω_r and ω_i are the real and imaginary parts, respectively, of the perturbation frequency) and $\omega_i \ll \omega_r$ within the limits of linear theory. Then in the case of high-frequency perturbations we can use $\omega_r^2 \sim \omega_{pe}^2(1+\lambda^2k^2)$ in Eq. (12) to obtain

$$A_{1y} \sim \frac{ck_y}{\omega_r} Q\phi_1, \quad (26)$$

where $Q=q_n\omega_{pe}^2/(2\gamma\omega_r+q_n\omega_{pe}^2)$. In terms of the magnetic field B_1 we can write this as

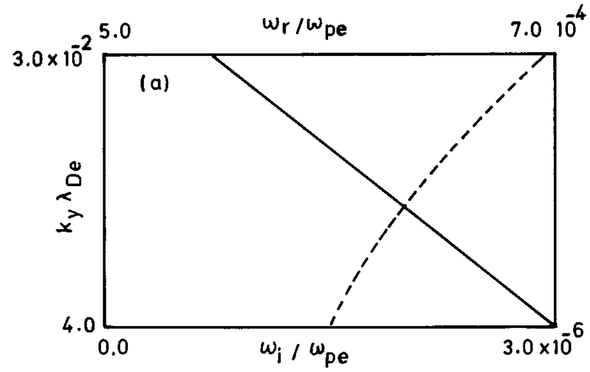
$$|B_1| \sim \frac{ckT_0}{e\omega_r}\frac{k_yk}{k_x}|Q|\left|\frac{e\phi_1}{T_0}\right|. \quad (27)$$

For the case of low frequency perturbations $\omega^2 \ll \omega_{pe}^2$ Eq. (12) can be approximated as

$$A_{1y} \sim i\frac{ck_yq_n}{S}\frac{T_0}{e}\frac{e\phi_1}{T_0}. \quad (28)$$

Again we can write this in terms of B_1 as

$$|B_1| \sim \frac{ck}{|S|}\left(\frac{k_yk}{k_x}\frac{T_0}{e}\right)|q_n|\left|\frac{e\phi_1}{T_0}\right|, \quad (29)$$



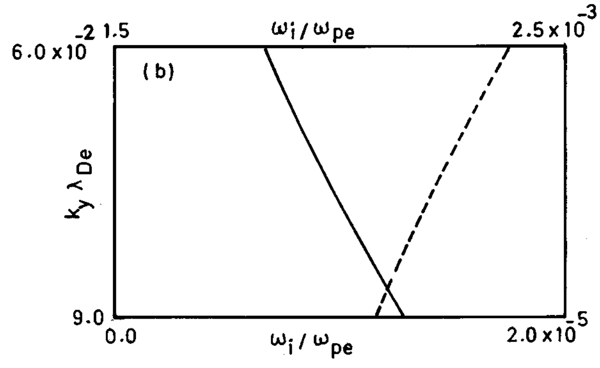


FIG. 2. The real ω_r (solid curve) and imaginary ω_i (dashed curve) frequencies of slow electromagnetic waves in electron-ion plasmas are plotted (a) for example 1 with $\kappa_n \sim 2\times10^3$ cm^{-1} and $\kappa_x \sim 1.6\times10^4$ cm^{-1}, (b) for example 2 with $\kappa_n \sim 2\times10^4$ cm^{-1} and $k_x \sim 4\times10^5$ cm^{-1}.

where $S=(\omega_r+i\omega_i)(-a+iq_n)$.

Let us consider the case of example 1 to see the order of magnitude of B_1 fields produced. The real and imaginary frequencies of the high-frequency unstable wave are found to be $\omega_r\sim 2.8\omega_{pe}$ and $\omega_i\sim 2 10^{-3}\omega_{pe}$, corresponding to $\kappa_n \sim 5\times 10^3$ cm^{-1} and $k_x\sim k_y\sim 4\times 10^4$ cm^{-1}. Then Eq. (27) gives

$$|B_1|\sim(1.8\times 10^4)\left|\frac{e\phi_1}{T_0}\right|. \tag{30}$$

Assuming the initial electrostatic perturbation to be about 1% of the background thermal energy, we obtain $|B_1|\sim 180$ G. If the growth rate is taken into account,

$$|B_1|\sim 180\exp(\omega_i t). \tag{31}$$

In $t=1\times 10^{-11}$ s this initial perturbation can grow to the magnitude of about 1 MG.

For a slowly growing B field we have $\omega_r\sim(3.2\times 10^{-4})\omega_{pe}$ and $\omega_i\sim(2.3\times 10^{-6})\omega_{pe}$, corresponding to $\kappa_n\sim 2\times 10^3$ cm^{-1}, $k_x\sim 1.6\times 10^4$ cm^{-1}, and $k_y\sim 4.3\times 10^4$ cm^{-1} in example 1. Then Eq. (29) gives

$$|B_1|\sim(5.4\times 10^5)\left|\frac{e\phi_1}{T_0}\right|. \tag{32}$$

It grows exponentially as

$$|B_1|\sim(5.4\times 10^5)\left|\frac{e\phi_1}{T_0}\right|\exp(\omega_i t). \tag{33}$$

We notice that as in the previous case for $|e\phi_1/T_0|=0.01$, $|B_1|\sim 1\times 10^6$ G in about a nanosecond. Similar results are obtained if we use the parameters of example 2.

V. DISCUSSION

It has been shown that linearly fluctuating electromagnetic fields can grow on both ion and electron time scales in an inhomogeneous unmagnetized plasma slab. A density gradient is required for the coupling of transverse and longitudinal modes which has been assumed to be maintained by external conditions. The electron temperature perturbation can generate a thermal wave having real frequency near that of the ion acoustic wave. Therefore, pure electron plasma models are not applicable for the study of such a slow perturbation. Moreover, it is found that such a thermal wave is not a pure transverse mode. The electron compressibility contributes to the electron temperature perturbation and hence the role of the electrostatic potential and the density fluctuation cannot be ignored. However, in a pure electron plasma the high-frequency longitudinal and transverse waves can couple in the presence of a density gradient as Eq. (21) shows. The two slow thermal modes are artificially coupled with the high-frequency waves in Eq. (24). The numerical solutions for both the plasma examples (1) and (2) show that the thermal wave is not a high-frequency electron wave with $\omega_{pi}\ll\omega$. Therefore we conclude that in a pure electron plasma only the high-frequency transverse wave can become unstable if $k_x\neq 0$.

To investigate the low-frequency magnetic perturbations we have to consider ion dynamics as well. Linear perturbations of an electron-ion plasma yield a sixth order polynomial in ω. There are two low-frequency and two high-frequency electromagnetic modes that remain stable if there is no propagation along the x axis. One slow and one fast electromagnetic modes can become unstable with a negative component of phase velocity $(v_{ph})_x<0$ in the x direction. The two conjugate modes with $(v_{ph})_x>0$ are damped. Since $\kappa_n>0$ is assumed, only the solutions with $(v_{ph})_x<0$ can become unstable, which corresponds to the situation $k_x<0$. That is, the modes propagating in the opposite direction to the density gradient for $\omega>0$ can grow due to plasma convection along the perturbation, giving rise to large magnetic fields. If we take into account the entropy increase, the problem becomes very interesting but complex. The density perturbation due to compressibility and the plasma vorticity can couple to generate these instabilities. In the presence of density inhomogeneity the electrons do not follow a Boltzmann distribution even on a slow time scale. In the presence of a density gradient the curl of the electric field is finite and the magnetic perturbations do not allow electrons to move freely, as they can under the influence of an electrostatic potential only. Therefore electron inertia becomes important. In this situation a slow electromagnetic unstable wave appears, which couples with the high-frequency transverse modes.

The solutions also show that one of the longitidinal plasma modes can grow. This unphysical result is due to the simplification of the fluid model which cannot take into account the effect of Landau damping. Landau damping can have an important effect on the instability criteria of electromagnetic modes as well, because electrostatic and magnetic vector potentials are coupled. Therefore a kinetic treatment would be very useful in the present investigation.

If the large growth rate of the high-frequency transverse wave predicted by fluid theory remains of the same order of magnitude in the kinetic treatment, then nonlinear effects should also be studied. A nonlinearly modulated electromagnetic wave may evolve slowly even in an electron plasma. Therefore we think that magnetic perturbations can grow on several temporal and spatial scales in an unmagnetized inhomogeneous plasma. Furthermore, in the linear limit the eigenvalue problem along the x direction should be solved with appropriate boundary conditions.

The present investigation also shows that any inhomogeneous plasma can support electromagnetic linear perturbations. Therefore, the seed of a magnetic field always exists in nonuniform plasmas. This may explain the initial magnetic field generation in space plasmas like planets, stars, etc.

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